Calc BC : Developing the Maclaurin Series (Pink sheet)

$$f(c) + \frac{f'(c)}{1!}x + \frac{f''(c)}{2!}x^2 + \frac{f'''(c)}{3!}x^3 + \dots$$

① $f(x) = \frac{1}{1-x} = (1-x)^{-1}$ $\quad f(c) = 1$

$f'(x) = 1(1-x)^{-2}$ $\quad f'(c) = 1$

$f''(x) = (2\cdot1)(1-x)^{-3}$ $\quad f''(c) = 2!$

$f'''(x) = (3\cdot2\cdot1)(1-x)^{-4}$ $\quad f'''(c) = 3!$

$f^4(x) = (4\cdot3\cdot2\cdot1)(1-x)^{-5}$ $\quad f^4(c) = 4!$

$$\frac{1}{1-x} = 1 + \frac{1}{1!}x + \frac{2!}{2!}x^2 + \frac{3!}{3!}x^3 + \dots$$

$$= 1 + x + x^2 + x^3 + \dots +$$ Yeah! Same as before!

② $f(x) = \frac{1}{1+x} = (1+x)^{-1}$ $\quad f(c) = 1$

$f'(x) = -1\cdot(1+x)^{-2}$ $\quad f'(c) = -1$

$f''(x) = (2\cdot1)(1+x)^{-3}$ $\quad f''(c) = 2!$

$f'''(x) = -(3\cdot2\cdot1)(1+x)^{-4}$ $\quad f'''(c) = -3!$

$f^4(x) = (4\cdot3\cdot2\cdot1)(1+x)^{-5}$ $\quad f^4(c) = 4!$

$$\frac{1}{1+x} = 1 - \frac{1}{1!}x + \frac{2!}{2!}x^2 - \frac{3!}{3!}x^3 + \frac{4!}{4!}x^4$$

$$= 1 - x + x^2 - x^3 + x^4 + \dots + \dots = \sum_{n=0}^{\infty}(-1)^n(x)^n$$

③ $\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \dots$

$\int_0^x \frac{1}{1+t}\,dt = \int_0^x 1 - t + t^2 - t^3 + t^4 \dots\,dt$

$\ln|1+x| = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \dots + = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$

④ $f(x) = e^x \quad f(0) = 1$

$f'(x) = e^x \quad f'(0) = 1$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

⑤ From Packet

$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

⑥ $\frac{d}{dx}(\sin x) = \frac{d}{dx}\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\right)$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$ (Same as on Packet!)

⑦ $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$

$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = 1 + (-x^2) + (-x^2)^2 + (-x^2)^3 + (-x^2)^4 + \dots$

$= 1 - x^2 + x^4 - x^6 + x^8 - x^{10} \dots$

⑧ on Pink sheet